QUARTERLY PROGRESS REPORT NO. 3
UNDER CONTRACT N00017-71-C-1417

to

Determine the Feasibility of Implementing Adaptive Echo Cancellation Techniques to Underwater Weapons

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I. INTRODUCTION

This is the third quarterly report issued under Contract N00017-71-C-1417. Work under THIS DOCUMENT the contract is aimed at determining the feasibility of implementing adaptive echo cancellation techniques to underwater weapons.

In work reported in progress reports 1 and 2, effort was aimed at determining the dynamic characteristics of the adaptive system represented in Fig. 1. In its major attributes this system is identical to one utilized by M. H. Sondhi[1] to cancel "echoes" generated at discontinuities in telephone communications lines. A more complex form of the system has been adopted for study within the last quarter, but the system described in Fig. 1 is a convenient vehicle for introductory remarks concerning the system's operation.

The function F(t) represents a received echo signal that is representible as a manifold of time shifted components. The components, in turn, are regarded as time shifted replicas of the reference function R(t).

An examination of Fig. 1 will show that the reference signal R(t) is admitted into a tapped delay line with uniform delay increment τ between consecutive taps. The output signal at each tap is therefore a time shifted replica of the transmit reference function and this signal is connected as one input to

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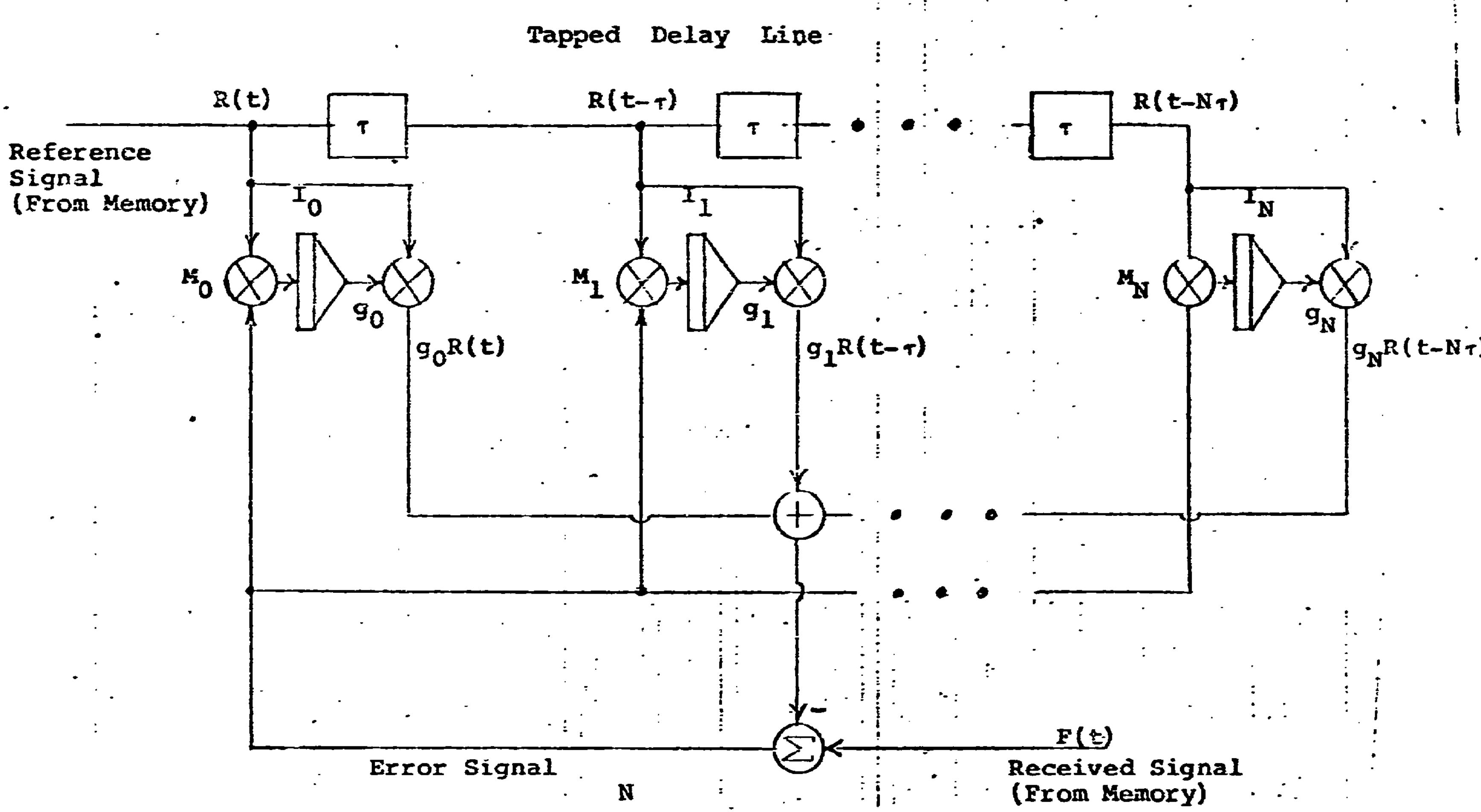
[1] Sondhi, M.H., An Adaptive Echo Canceller, B.S.T.J. 46, March 1967, pp. 497-511



$$\varepsilon(t) = F(t) - \sum_{k=0}^{N} g_k R(t-k\tau)$$

Fig. 1. Simplified Diagram of an Adaptive Echo Canceller

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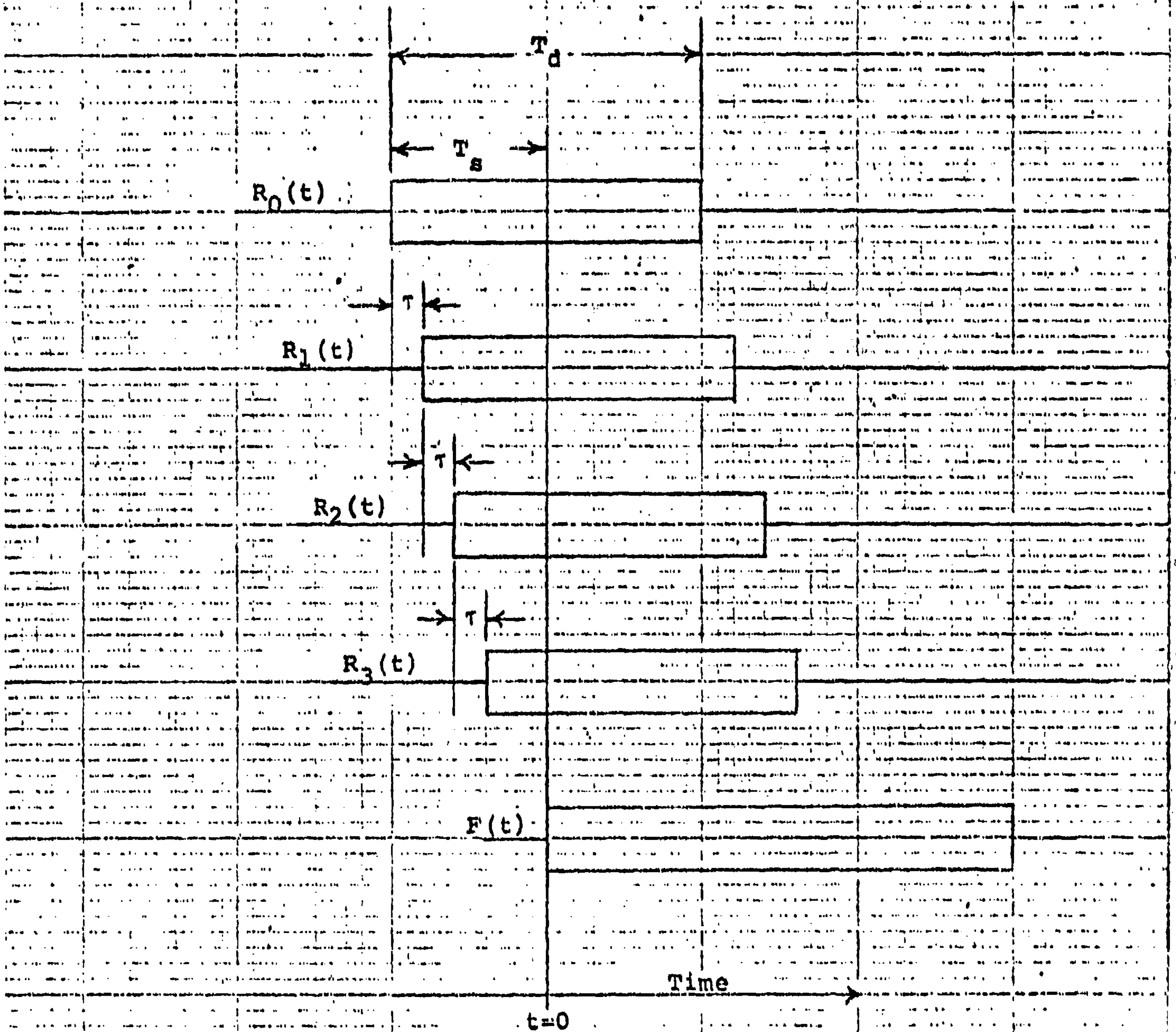
a multiplier. The second input to the multiplier is an error signal $\epsilon(t)$, that is common to all the multipliers in the system. The output of any one multiplier forms the input to an integrator, and an examination of Fig. 1 will show that the multiplier-integrator components implement a cross-correlation between the delayed reference signals $R(t-n\tau)$ and the common error signal $\epsilon(t)$. The output of the integrator is admitted as a control voltage to a second multiplier unit g_n. The delayed reference signal is connected as a second input to this multiplier and the output of the unit can be expressed as $g_n R(t-n\tau)$. The gain parameters g_n can take on positive or negative values and can also take on the value zero. As is indicated in Fig. 1, the value of g_n is controlled by its associated cross-correlator. The output signals of the correlator units are summed together and this composite signal is subtracted from the received signal, $F(t)$, to form the error signal $\epsilon(t)$. Thus, under ideal circumstances, the correlators provide a measure of the time registry between their reference voltage outputs and the components of the echo signal and control the gain parameters to null the error signal. The study of the system's response under less than ideal conditions is a main goal of the work in progress.

The intent is to use the processor to implement what may be termed as a steady state mode of operation. This mode of operation requires that each of the signals taken from the tapped delay line be in an active state over the whole time interval of a given test. This requirement imposes constraints on the time relationship between $F(t)$ and the reference signals, and also stipulates a maximum time duration for a given test.

The onset of $F(t)$ will define the zero of time as is indicated in Fig. 2. The requirement that all of the delayed reference signals be in an active state during the time span



(U) Fig. 2. Time Synchronism of a Four Loop System

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of a test implies that the reference functions be in an active state before the onset of F(t). Moreover, examination of Fig. 2 will show that the time interval over which steady state tests can be conducted is specified by

$$0 \le t \le T_d - T_s \ . \tag{1}$$

By inspection of Fig. 1, the error signal can be described by

$$\epsilon(t) = F(t) - \sum_{k=0}^{N-1} g_k(t) R_k(t) \ , \tag{2}$$

where

$$R_k(t) = R(t-k\tau) \ . \tag{3}$$

In addition, an examination of Fig. 1 will show that for each of the integrator-multiplier units one can write an equation of the form

$$g_n(t) = g_n(0) + \frac{1}{T}\int_0^t R_n(x)\epsilon(x)\,dx \ . \tag{4}$$

The system's feedback is made more apparent by taking the time derivative of Eq. (4) and writing out the full expression for the error function $\epsilon(t)$. This results in N equations of the form

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$$\dot{g}_n(t) = \frac{R_n(t)}{T}[F(t) - \sum_{k=0}^{N-1} g_k R_k] \quad . \qquad (5)$$

In the last progress report, numerical techniques were described that permitted a direct integration of the coupled equations represented by Eq. (5). The computer program, as described in the second progress report entailed the following operations:

1. A first phase of the program was concerned with obtaining accurate values of the gain parameters at four consecutive points in time through a process of iteration,

2. A second phase of the program was concerned with continuing the solution through a process of extrapolation.

Early exercises of the program clearly indicated that the extrapolation technique did not provide stable solutions to the coupled equations represented by Eq. (5). To remedy this defect an additional iterative operation was required after step 2, as described above. With this modification the program operated satisfactorily and was used to demonstrate to this program's sponsor the computed response of the system for a number of cases. These cases are touched on briefly in the following section of the report. However, an additional deficiency became apparent, this time relating to the configuration of the system described in Fig. 1. The system, as described, is adequate only for the investigation of those cases in which the signal components are in exact time registry with a subset of the reference signals taken from the tapped delay line.

Accordingly, agreement was reached with the program sponsor to generalize the configuration of the system through the adoption of a Quadrature Loop system. Description of this system is deferred to a later section of this report. However, investigation relating to the new system finally entailed the development of a new computer program. This program is again based on iterative techniques, but differs substantially in detail from the original program developed for use with the system represented in Fig. 1. Moreover, through slight modification, the new program can be used for investigation of either the old or the new system configurations. Accordingly, further discussion of the program described in the second progress report will be omitted. Finally, results computed with the new program are presented and discussed in the following pages of the report.

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II. TECHNICAL PROGRAM

A. RESULTS COMPUTED ON THE BASIS OF SONDHI's SYSTEM

Before addressing the Quadrature Loop system, a number of results computed for the system configuration shown in Fig. 1 will be presented. The results in question are solutions of the coupled gain equations represented by Eq. (5) and were obtained through use of a modified form of the computer program described in the second progress report.

Figure 3 is a plot of the gain versus time variation for a single loop system in which the received signal is taken as being identical to the reference signal and with both signals occurring in perfect time registry. The case in point is useful in demonstrating that each of the system loops can be regarded as having the frequency response characteristics of a single pole, low-pass filter.

For a single loop, Eqs. (5) take the form

$$\mathring{g}_0 = \frac{R_0}{T}[F - g_0 R_0] \quad . \qquad (6)$$

Through a rearrangement of terms and the introduction of an integrating factor, Eq. (6) can be written as

$$\frac{d}{dt}\{g_0\, e^{\frac{1}{T}\int_0^t R_0^2(\eta)\,d\eta}\} = \frac{R_0 F}{T}\, e^{\frac{1}{T}\int_0^t R_0^2(\eta)\,d\eta} \quad . \qquad (7)$$

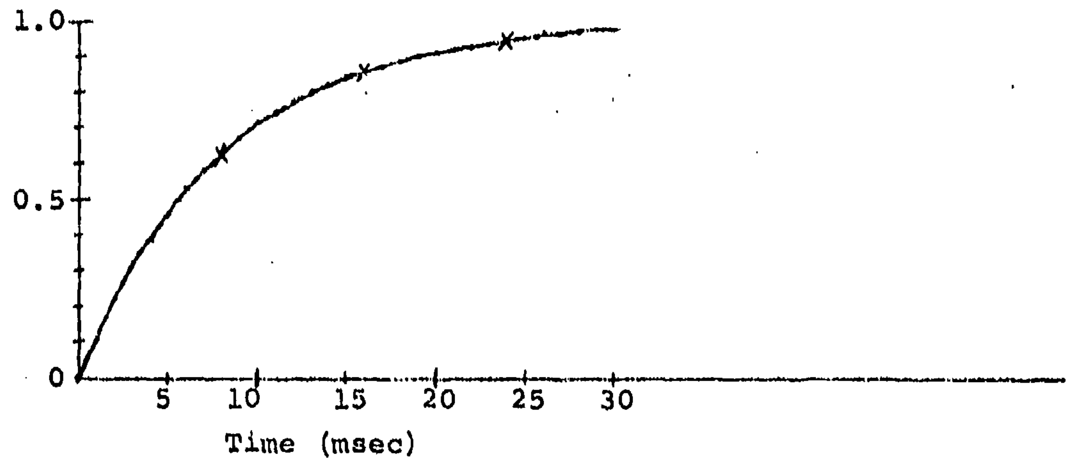

Fig. 3. Gain Versus Time for Single Loop, Sondhi Type System, Responding to Single Component Signal of Zero Doppler with Perfect Time Registry. Received and Reference Signals are Linear FM Slides of 0.064 Sec. Duration, 1 kHz Bandwidth, Start Frequency of 0.5 kHz. Integrator Time Constant is 4 msec.

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Equation (7) can then be integrated to yield

$$g_0(t) = g_0(0)\, e^{-\frac{1}{T}\int_0^t R_0^2(\eta)\,d\eta} + \frac{1}{T}\int_0^t R_0(x)F(x)\, e^{-\frac{1}{T}\int_x^t R_0^2(\eta)\,d\eta}\, dx \tag{8}$$

Because $R_0(x)$ is an FM slide, it can be represented in the form

$$R_0(x) = \cos\,\theta_0(t) \quad , \tag{9}$$

where

$$\theta_0(t) = \omega_0 t + \frac{\mu}{2}t^2 \tag{10}$$

In Eq. (10), ω_0 represents the beginning radian frequency of the FM slide and μ is a measure of the signal's bandwidth. Because of the trignometric form of the right hand side of Eq. (9), the square of the function can be expressed as

$$R_0^2(\eta) = \frac{1}{2} + \frac{1}{2}\cos\,2\theta_0(\eta) \quad . \tag{11}$$

The second term on the right hand of Eq. (11) represents the second harmonic of the FM slide. Because of the oscillatory nature of this term it will be assumed that the time integral of $R_0^2(\eta)$ can be approximated by the integral of the constant term, $\frac{1}{2}$, and that the integral of the second term on the right in Eq. (11) can be ignored. With this approximation Eq. (8) can be written in the form

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$$g_0(t) \simeq g_0(0)\, e^{-\frac{t}{2T}} + \frac{1}{T}\int_0^t e^{-\frac{1}{2T}(t-x)} R_0(x)F(x)\,dx \quad . \tag{12}$$

Examination of the second member on the right in Eq. (12) will show that the term has the form of a convolution integral. The same expression describes the output of a filter that has an impulse response, $e^{-\frac{t}{2T}}$, and an input signal equal to the product $R_0(x)F(x)$ along with an impulse function to generate the term $g(0)$. A filter of this type can be realized with a series connected RC network with the input signal applied to the resistor and the output signal taken across the capacitor. The realization requires only that the RC product be equal to 2T. Thus, if the filter's low frequency cut-off is designated by f_c, then

$$f_c = \frac{1}{2\pi RC} = \frac{1}{4\pi T} \quad . \tag{13}$$

If $F(x)$ is taken equal to $R(x)$, and if the product is again approximated by the factor $\frac{1}{2}$, then substitution in Eq. (12) yields

$$g_0(t) = g_0(0)\, e^{-\frac{t}{2T}} + [1 - e^{-\frac{t}{2T}}] \quad . \tag{14}$$

As a test of the approximations leading to Eq. (14), the computer program was used to compute $g_0(t)$ for a case in which $g_0(0)$ was zero, $F(t)$ was equal to $R_0(t)$, with $R_0(t)$ described by Eqs. (9) and (10). In addition, the beginning frequency of the FM slide

was taken at 500 Hz, and the signal's bandwidth was taken as 1000 Hz over a 64 msec duration. The integrator time constant T was set equal to 4 msec, and the computed results are presented in graphical form in Fig. 3 . The results obtained with the computer are plotted as a solid curve while points evaluated from Eq. (14) are represented by the symbol x along the curve. The scale to which the graph is drawn does not permit a visual distinction between the values arrived at by the two methods described, and a more careful examination of numerical data indicates that the deviation between the results obtained with the computer versus those computed from Eq. (14), is on the order of 0.05 percent. Thus, the approximations and conclusions entailed in the development of Eq. (14) are considered valid.

The computer program developed for the investigation of Sondhi's system was used to explore a number of cases, and certain of those cases were run with the project sponsor in attendance. Because current interest relates to a modified form of the system, computed results for only one additional case involving Sondhi's system will be presented in this report. The case involves a four-loop system responding to a three-component signal. The components were considered to have levels that were respectively 8, 4, and 2 times the levels of the system's reference functions. Moreover, the components were considered to be unaltered by Doppler and occurred in perfect time registry with the first three corresponding reference signals of the system. The computed results are presented in graphical form in Fig. 4. The main point in presenting these data is to show that mutual interference occurs between the loops of Sondhi's system. This action is evidenced by a high frequency ripple superimposed on a slowly varying component associated with each of the gain parameters. The high frequency component is evident in the drawing's

of Fig. 4 , and is attributed to a given loop's response to a signal component not in time registry with the reference function associated with the loop. However, Fig. 4 shows that for the case considered, the system still proceeds to a null solution although not in a monotonic manner. A similar phenomenon is observed with a modified system described later in this report, and to avoid repetition, further discussion of this phenomenon will be deferred pending description of the newer system.

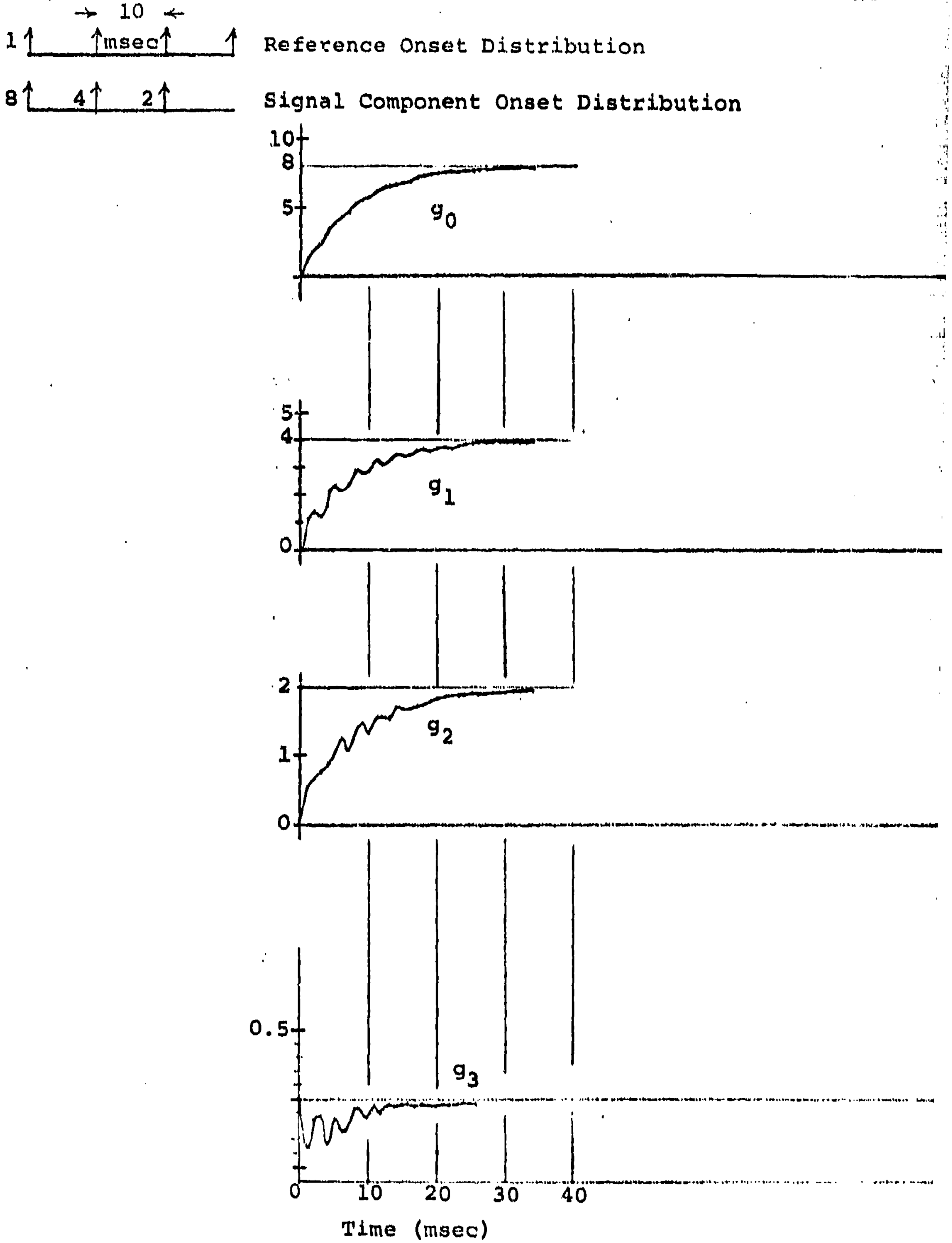

Fig. 4. Gain Parameters Versus Time for Four Loop, Sondhi Type Syst Responding to Three Component Signal with 8, 4, 2 Amplitude Distribution. Data Shown Are for Zero Doppler and Perfect Time Registry. System and Signal Parameters Are as Cited i Fig. 3.

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B. THE QUADRATURE LOOP SYSTEM

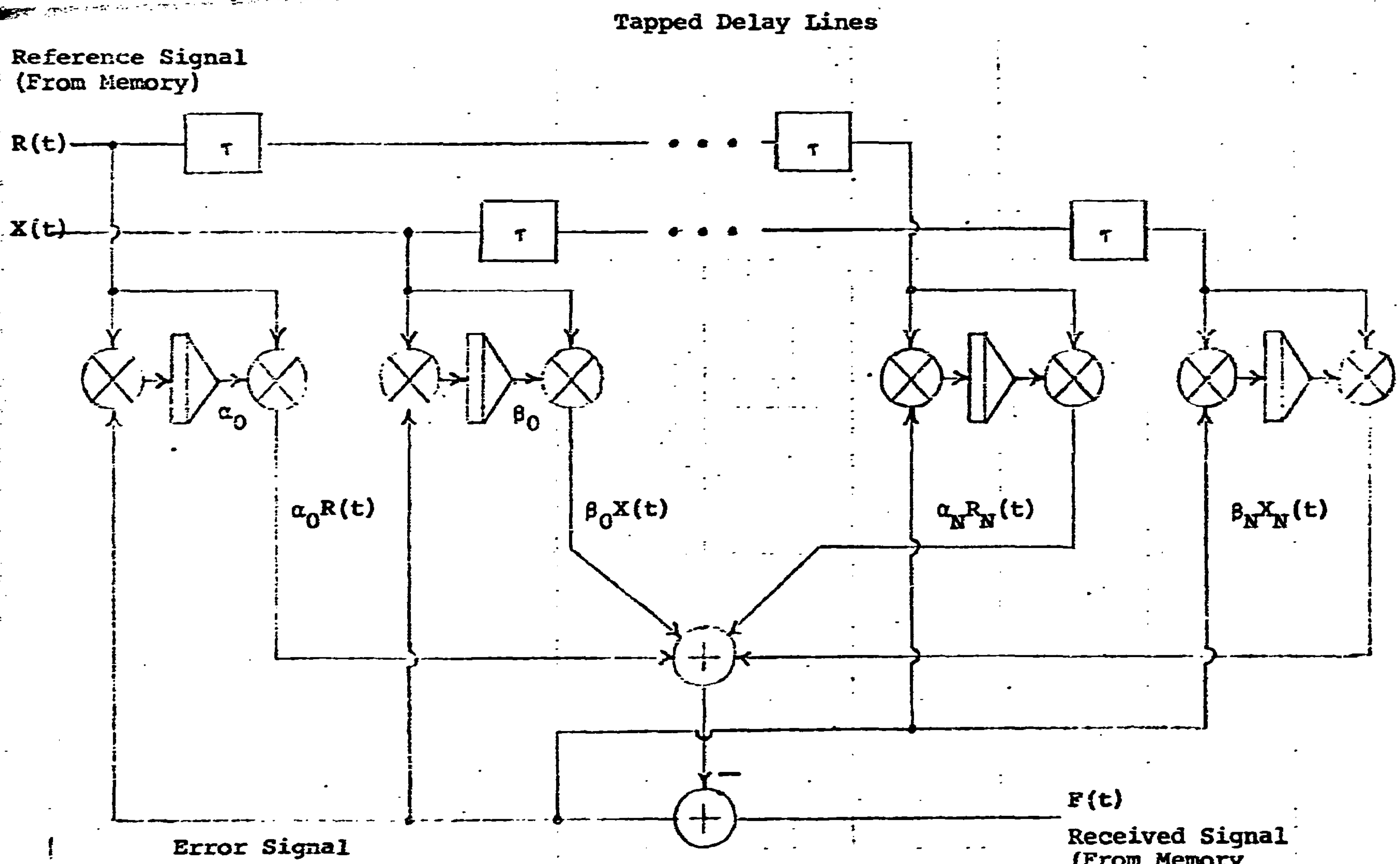
Owing to the fact that the echo canceller's error signal nulls through a subtractive process, a precise alignment in time must be maintained between a subset of the reference signals and the various components of the echo in order to achieve a nulling action in Sondhi's system. Thus, in practice, misalignments in time registry on the order of one fourth period at the center frequency of the FM signals would be enough to prevent the nulling action, because this situation would be roughly equivalent to subtracting two sinusoids characterized by a relative phase shift - a process which also cannot produce a null that is uniform with time. Accordingly, throughout the last quarter, studies have been based on a modified form of Sondhi's system that incorporates new degrees of freedom to enable an adaptive form of phase compensation. The modified echo canceller is referred to as a Quadrature Loop system throughout this report and its configuration is described by Fig. 5.

As can be inferred from a comparison of Figs. 5 and 1 the quadrature loop system differs from Sondhi's in that two reference functions are supplied to the system and each loop is provided with two separate paths. The reference function R(t) is expressed as before by

$$R(t) = \cos(\omega_0 t + \frac{\mu t^2}{2}) \quad , \tag{15}$$

while the function X(t) is the quadrature component

$$X(t) = \sin(\omega_0 t + \frac{\mu t^2}{2}) \quad . \tag{16}$$



$$\epsilon(t) = F(t) - \sum_{k=0} [\alpha_k R(t-k\tau) + \beta_k X(t-k\tau)]$$

Fig. 5. Adaptive Echo Canceller Employing Quadrature Loops

The two functions are represented in Fig. 5 as being transmitted along separate tapped delay lines. The signals taken after n delay elements can therefore be represented as

$$R_n(t) = R(t-n\tau) \text{ , and} \tag{17}$$

$$X_n(t) = X(t-n\tau) \text{ .} \tag{18}$$

At each such location two adaptive gain control units similar to those employed in Sondhi's system scale the signal levels transmitted through the loops. Thus, the contribution to a central summation point made by the quadrature loop energized after n delay elements can be expressed as

$$S_n(t) = \alpha_n R_n(t) + \beta_n X_n(t) \quad . \tag{19}$$

The mechanism through which adjustment of the gains α_n, β_n can result in an adjustment of phase angle can be most readily demonstrated by first expressing the gain parameters in terms of auxiliary quantities C_n, Φ_n, through the equations

$$\alpha_n(t) = C_n(t) \cos \Phi_n(t) \tag{20}$$

$$\beta_n(t) = C_n(t) \sin \Phi_n(t) \quad . \tag{21}$$

Then expansion of Eq. (19) yields

$$S_n(t) = C_n(t)[\cos \Phi_n(t) \cos[\omega_0(t-n\tau) + \frac{\mu}{2} (t-n\tau)^2]$$

$$+\sin \Phi_n(t) \sin[\omega_0(t-n\tau) + \frac{\mu}{2} (t-n\tau)^2]] \quad . \tag{22}$$

Equation (22) can be recognized as being trignometrically identical to

$$S_n(t) = C_n(t) \cos[\omega_0(t-n\tau) + \frac{\mu}{2} (t-n\tau)^2 - \Phi_n(t)] \quad . \tag{23}$$

Equation (23) of course, provides no information concerning the system's adaptive response. Nevertheless , considerable insight can be gained with respect to the computer generated data presented in a later section of this report by first considering a few elementary but highly hypothetical cases.

As a first case, consideration will be directed toward a single quadrature loop system responding perfectly to a single component echo but with the echo component misaligned in time with the reference signals supplied by the tapped delay line.

For convenience, take the quadrature loop signal as

$$S_0(t) = C_0(t) \cos[\omega_0 t + \frac{\mu}{2}t^2 - \Phi_0(t)] \quad , \tag{24}$$

and take the echo signal in the form

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$$F(t) = \cos[\omega_0(t-\epsilon) + \frac{\mu}{2}(t-\epsilon)^2]$$

$$= \cos[(\omega_0 t + \frac{\mu}{2}t^2) - (\mu\epsilon t + \omega_0\epsilon - \frac{\mu}{2}\epsilon^2)] \quad . \tag{25}$$

The second expression on the right hand side of Eq. (25) is arrived at by a simple rearrangement of terms. However, comparison of this expression with Eq. (24) will show that the two can be made identical by setting

$$\phi_0(t) = \mu\epsilon t + \omega_0\epsilon - \frac{\mu}{2}\epsilon^2 \quad , \tag{26}$$

$$C_0(t) = 1 \quad . \tag{27}$$

Thus, by Eqs. (20) and (21), the gain parameters for the loop would be

$$\alpha_n = \cos(\mu\epsilon t + \omega_0\epsilon - \frac{\mu}{2}\epsilon^2) \tag{28}$$

$$\beta_n = \sin(\mu\epsilon t + \omega_0\epsilon - \frac{\mu}{2}\epsilon^2) \quad . \tag{29}$$

Thus, in a perfect system of the type considered, the gain parameters would describe a sinusoidal variation with time with a radian frequency equal to $\mu\epsilon$. One can therefore see that the frequency of the variation increases directly with the time misalignment ϵ.

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As a second case, consider a single component echo with Doppler that is in exact time registry with a single quadrature loop but with reference functions that do not reflect Doppler. For this case take

$$S_0(t) = C_0(t) \cos[\omega_0 t + \frac{\mu}{2}t^2 - \Phi_0(t)] \quad , \qquad (30)$$

and assume the received signal given by

$$F(t) = \cos[\alpha\omega_0 t + \mu\alpha^2\frac{t^2}{2}] \quad . \qquad (31)$$

In Eq. (31) the effects of Doppler are reflected by applying a constant scale factor α to the time parameter. By equating the arguments of the right hand functions of Eqs. (30) and (31), one can readily show that the two equations are identical if

$$\Phi_0(t) = \omega_0(1-\alpha)t + \frac{\mu}{2}(1-\alpha^2)t^2 \qquad (32)$$

$$C_0(t) = 1 \qquad (33)$$

Equation (32) shows that the gain parameters for this case would describe a linear FM variation with time.

As noted previously, the two cases just considered do not actually describe the dynamic response of the system. A computer program has been utilized to demonstrate system response

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for a number of cases. Referring to Fig. 5 , the gain parameter in any loop is the integral of the product of the corresponding reference signal and the error signal. Thus, the gain parameters as a function of time can be inferred by solving the coupled linear differential equations specified by

$$\mathring{\alpha}_m = \frac{R_m}{T}\{F(t) - \sum_k [\alpha_k R_k + \beta_k X_k]\} \quad , \tag{34}$$

$$\mathring{\beta}_m = \frac{X_m}{T}\{F(t) - \sum_k [\alpha_k R_k + \beta_k X_k]\} \quad . \tag{35}$$

The computer program in question implements a direct numerical integration of the coupled equations represented by Eqs. (34) and (35).

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C. BRIEF DESCRIPTION OF THE COMPUTER PROGRAM USED FOR INVESTIGATION OF THE QUADRATURE LOOP SYSTEM

The computer program developed for use in an investigation of the Quadrature Loop system resembles, in many respects, the program developed for a study of Sondhi's system. This being the case, a detailed description of the new program will be deferred to the final report. The salient features of the new program are outlined below.

The operation of the program is based on numerical methods of solution derived in the second progress report. Only the results of those derivations will be presented here, and are expressed by the following equations:

$$\alpha_m(\gamma+h) = \alpha_m(\gamma) + \frac{h}{12T}\{5R_m(\gamma)\varepsilon(\gamma) + 8R_m(\gamma+h)\varepsilon(\gamma+h) - R_m(\gamma+2h)\varepsilon(\gamma+2h)\} \qquad (36)$$

$$\beta_m(\gamma+h) = \beta_m(\gamma) + \frac{h}{12T}\{5X_m(\gamma)\varepsilon(\gamma) + 8X_m(\gamma+h)\varepsilon(\gamma+h) - X_m(\gamma+2h)\varepsilon(\gamma+2h)\} \qquad (37)$$

$$\alpha_m(\gamma+2h) = \alpha_m(\gamma) + \frac{h}{3T}\{R_m(\gamma)\varepsilon(\gamma) + 4R_m(\gamma+h)\varepsilon(\gamma+h) + R_m(\gamma+2h)\varepsilon(\gamma+2h)\} \qquad (38)$$

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$$\beta_m(\gamma+2h) = \beta_m(\gamma) + \frac{h}{3T}\{X_m(\gamma)\epsilon(\gamma) + 4X_m(\gamma+h)\epsilon(\gamma+h) + X_m(\gamma+2h)\epsilon(\gamma+2h)\} \quad . \tag{39}$$

In Eqs. (36) through (39), α_m, β_m are the gain parameters, R_m, X_m are the reference functions, and T is the integrator time constant. Moreover, γ is a particular value of time and h is a small time increment. The error function is given by

$$\epsilon(\gamma) = F(\gamma) - \sum_k [\alpha_k(\gamma)R_k(\gamma) + \beta_k(\gamma)X_k(\gamma)] \quad , \tag{40}$$

where $F(\gamma)$ is the received signal at $t=\gamma$.

Input to the program includes initial values for the gain parameters at t=0. The first phase of the program is aimed at finding values of the gain parameters at t=h and t=2h. This is carried out through a process of iteration involving Eqs. (36) through (39) with γ set equal to zero. On the right hand sides of these equations, the iterations relate to the terms $\epsilon(h)$ and $\epsilon(2h)$. The process commences by providing initial estimates for $\alpha_k(h)$, $\alpha_k(2h)$, $\beta_k(h)$, and $\beta_k(2h)$. Then through use of Eq. (40) initial estimates of $\epsilon(h)$ and $\epsilon(2h)$ are computed. Iteration then commences by computing a new value for $\alpha_0(h)$ from Eq. (36) after which the initial estimate of $\epsilon(h)$ is corrected to reflect the new value of $\alpha_0(h)$. The same process continues for $\beta_0(h)$, $\alpha_0(2h)$, $\beta_0(2h)$ and commences for $\alpha_1(h)$. One iteration is complete after the process has propagated through $\beta_{N-1}(2h)$, where N is the number of Quadrature Loops. A test is then applied to determine the

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error between successive iterations. Thus, if n in ${}_n\alpha_k(h)$ designates the n^{th} iteration, the error is specified by

$$E_1 = \sum_{k=0} \sum_{m=1} |{}_{n+1}\alpha_k(mh) - {}_n\alpha_k(mh)| + |{}_{n+1}\beta_k(mh) - {}_n\beta_k(mh)| \quad . \tag{41}$$

During each iteration a new value for E_1 is computed and the iterative process continues until E_1 is less than a maximum value, typically taken as 10^{-10}. In most cases the error E_1 falls below this maximum during five iterations carried out for a four loop system.

After refined values become available for the gain parameters at t=0, t=h, and t=2h, the program implements an iterative process based on Eqs. (38) and (39) alone. On the right hand side of these equations, the iterative process relates solely to the term $\epsilon(\gamma+2h)$, The process is again initiated by providing an estimate for the gain parameters at the time γ+2h. The convergence rate of the iterative process is greatly improved by solving Eqs. (38) and (39) explicitly for $\alpha_m(\gamma+2h)$, $\beta_m(\gamma+2h)$ on the assumption that parameters of index other than m have the correct value. For example, in Eq. (38), this consists of solving for ${}_{n+1}\alpha_m(\gamma+2h)$ in

$$[1 + \frac{h}{3T} R_m^2(\gamma+2h)]_{n+1}\alpha_m(\gamma+2h)$$

$$= \alpha_m(\gamma) + \frac{h}{3T} [R_m(\gamma)\epsilon(\gamma) + R_m(\gamma+h)\epsilon(\gamma+h)$$

$$+ R_m(\gamma+2h)\epsilon_m(\gamma+2h)] \quad , \tag{42}$$

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where

$$\epsilon_m(\gamma+2h) = \epsilon(\gamma+2h) + {}_n\alpha_m(\gamma+2h)R_m(\gamma+2h) \quad . \qquad (43)$$

The process represented by Eqs. (42) and (43) acts to remove the dependence on $\alpha_m(\gamma+2h)$ from the right hand side of Eq. (32).

The reference functions and the received signal functions are generated within the program through use of signal generator subroutines. In cases involving Doppler, the time parameter within these subroutines is appropriately scaled to reflect assumed radial speed components.

In the cases considered in the following section of the report computation was carried out through use of a Data General minicomputer incorporating a software "multiply-divide" subroutine and reading out via a standard teletype unit. The starting frequency of the FM signals was taken to be 750 Hz and the frequency was assummed to slide through 500 Hz in a 128 msec time interval. For these computations the increment h was taken to be 50 microseconds, and experience indicates that extending the range of the integrations over one millisecond requires approximately 40 seconds of computation followed by about 20 seconds of readout. Therefore a typical run requires in excess of 1½ hours for execution.

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D. COMPUTED RESPONSE OF A FOUR CHANNEL, QUADRATURE LOOP SYSTEM FOR SEVERAL TYPES OF INPUT SIGNALS

1. Parameters of The System

The computed results presented in the following pages of this report were based on a system incorporating four quadrature loops and three pairs of delay elements. Throughout the runs the time delays brought about by the delay elements was assumed to be 10 msec. The integrator time constant T was taken to be 4 msec.

The FM signals used in the computations were assumed to commence at an instantaneous frequency of 750 Hz and to increase linearly with time up to 1250 Hz in a time interval of 128 msec. Thus, the signals were assumed to vary over a 500 Hz band in 128 msec. The integration interval, h, was set equal to 50 μsec throughout the computation. In all cases the peak level of the reference signals were assumed to establish a unit level, and the levels cited for the components of received signals are their peak levels specified in terms of unit levels.

The data are presented in the form of manually plotted graphs of the gain parameters versus time. The graphs were drawn from a data printout executed at 1 msec intervals. Conversion of the printed data into graphical form proved a very laborious procedure and this fact limited the number of cases that could be included in this report.

Finally, a comment concerning the components of the received signal is in order. Each component is considered made up only of a time shifted cosine dependence such as characterizes the reference functions $R_k(t)$. Consequently, in cases involving

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exact time registry between undoppled signal components and certain of the reference signals, an exact null condition requires that the gain parameters β_k should be zero.

2. Computed Results

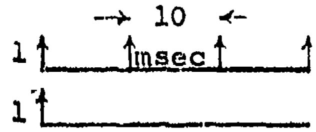
a. Case 1

For this case the assumption was made that a single component of zero Doppler and unit level occurred in exact time registry with the reference signal $R_0(t)$. Fig. 6 presents the computed results.

Because of the time coincidence between the signal and the reference function $R_0(t)$, a true null condition requires that α_0 be unity and all other gain parameters be zero. The manner in which the condition is approached is illustrated in Fig. 6 . The top graph in Fig. 6 shows the response of α_0, β_0 as a function of time. It is perhaps well to stress that the inflections in these curves are not the result of a trembling hand in the graphing process but rather the result of interaction among the loops.

The nominal behavior of the system appears to be as follows:

(1) Because R_0 is in time coincidence with the signal, the product of the two signals includes a d.c. term, which on being integrated causes α_0 to rise in the positive direction, and to introduce a nulling signal into the error channel. However, before the null can be completed the error channel still retains a substantial level of the received signal and the data suggest that during this time the three remaining channels act to participate in the nulling action.



Reference Onset Distribution

Signal Component Onset Distribution

RUN CONDITIONS:

Single component signal of unit amplitude, perfect time registry with $R_0(t)$.

Doppler = 0
T = .004 sec

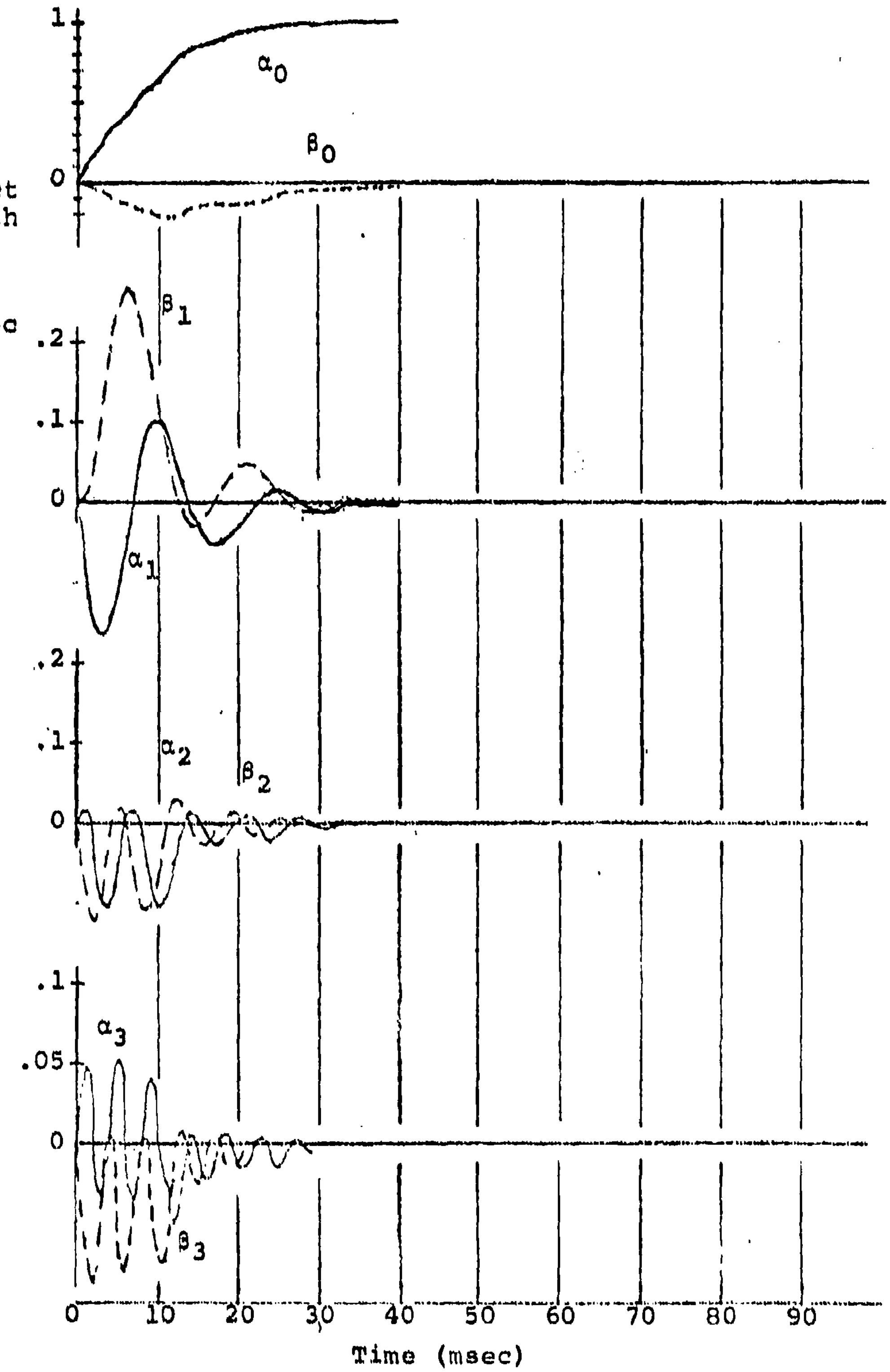


Fig. 6. Gain Parameters Versus Time for a Four Channel, Quadrature Loop System. Case 1

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In the description of the Quadrature Loop System (Section II-B) it was pointed out that in an idealized system a loop that is out of time registry with an FM signal by a time ϵ could still act to null the signal by describing sinusoidal variations of frequency $\frac{\mu\epsilon}{2\pi}$. For the case at hand

$$\frac{\mu}{2} = 2\pi\left(\frac{500}{.128}\right)\sec^{-2} \quad , \tag{44}$$

and ϵ is a multiple of 10 msec. Thus, the required frequency of variation for ϵ=.01 sec is

$$\frac{\mu\epsilon}{2\pi} \simeq 78.2 \text{ Hz} \quad , \tag{45}$$

corresponding to a period of 12.8 msec. In a nominal way, the variations of α_1 and β_1 suggest the incorporation of frequency components of this period in their makeup. Moreover, the frequency of variation appears to increase appropriately for α_2, β_2 and α_3, β_3 to reflect their respective degrees of misalignment with the received signal. An examination of Fig. 6 makes clear that the gain parameters for other than the zeroth loop cannot be represented exclusively as sinusoids or even as sinusoids multiplied by a slowly decreasing function such as an exponential. The parameter β_1 appears to be offset in the positive direction, while α_2, β_2, and β_3 are offset in the negative direction. A simple explanation of the noted offsets is not possible at the present time. However, the offsets appear to consist of functions of relatively slow variation. One is tempted to speculate that

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the slowly varying offset quantities could admit sufficient levels of the various reference functions into the error channel to account for the inflections shown by α_0 and β_0.

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b. Case 2

For this case the received signal was assumed to consist of three components, with amplitude distributions of 8, 4, 2 and with these components in exact time registry with the reference functions $R_0(t)$, $R_1(t)$, and $R_2(t)$ respectively. Doppler was assumed to be zero and the integrator time constant, T, was assumed to be 4 msec.

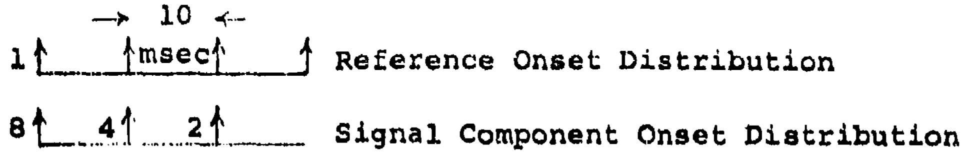
In this case again the system converged to a valid solution, and the values of the gain parameters versus time are indicated in Fig. 7 . Interaction among the loops is again suggested by the data and this is again attributed to a tendency for each loop to respond to all of the components of the received signal. In general, the parameters achieve stable and accurate values only after the error signal has been tightly nulled.

c. Case 3

The received signal for this case was assumed to consist of a single component of unit level whose onset time occurred 5 msec after the onset of $R_0(t)$ and 5 msec before the onset of $R_1(t)$. Doppler was assumed to be zero and the integrator time constant T was taken equal to 4 msec. The response of the system to this type of signal is represented in Fig. 8.

In the case under consideration, the system would not null the error signal. The failure of the system to achieve a null is attributed to phase shifts introduced into the gain parameter signals. The phase shifts in turn appear to be caused by the integrators in conjunction with the feedback paths in each loop.

The signal supplied to the error channel by the initial quadrature loop can be expressed in the form



RUN CONDITIONS:

Received signal made up of three components with 8, 4, 2 amplitude distribution. Components in perfect time registry with R_0, R_1, R_2, respectively.

Doppler = 0
T = .004 sec

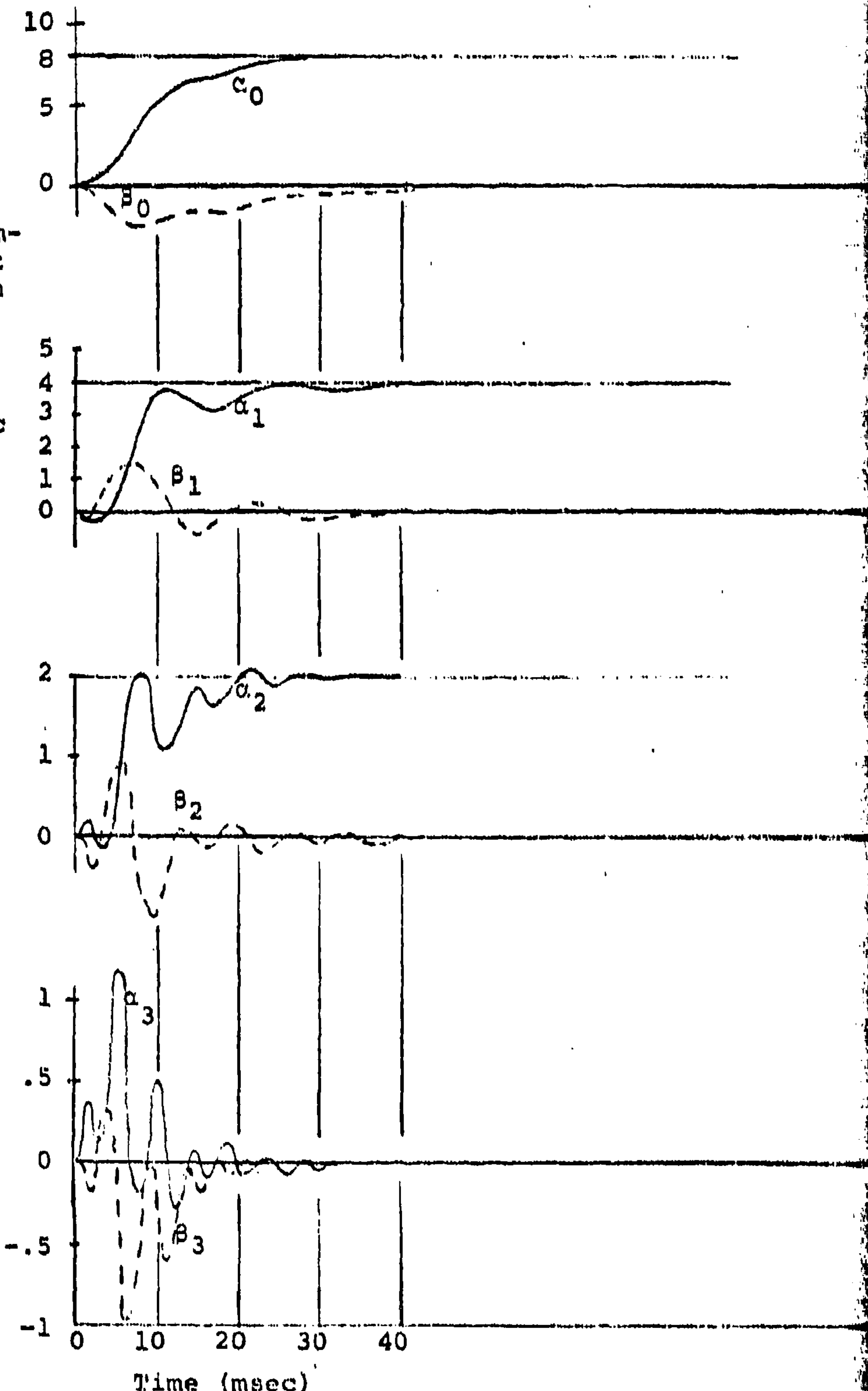
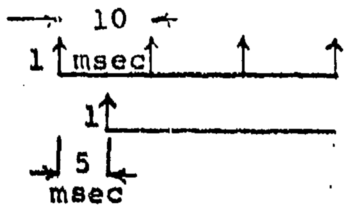


Fig. 7. Gain Parameters Versus Time for a Four Channel, Quadrature Loop System. Case 2



Reference Onset Distribution

Signal Component Onset Distribution

RUN CONDITIONS:

Single component signal of unit amplitude, with onset 5 msec after that of $R_0(t)$

Doppler = 0

T = .004 sec

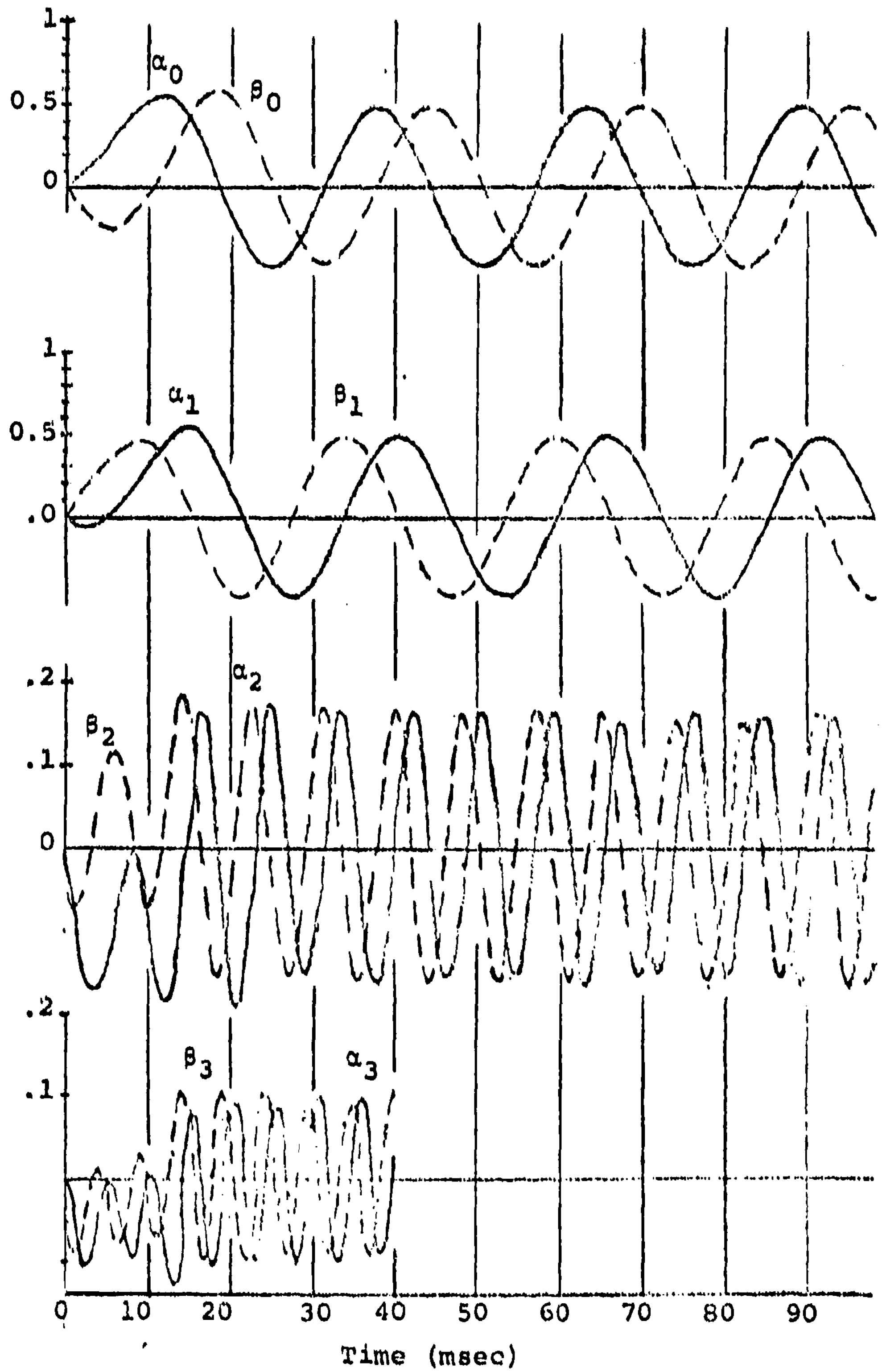


Fig. 8. Gain Parameters Versus Time for a Four Channel Quadrature Loop System. Case 3

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$$C_0 = \alpha_0 \cos[\omega_0(t+T_2) + \frac{\mu}{2}(t+T_2)^2]$$

$$+ \beta_0 \sin[\omega_0(t+T_2) + \frac{\mu}{2}(t+T_2)^2] \quad . \tag{46}$$

The single component signal is specified by

$$S_0 = \cos[\omega_0(t+T_2-\epsilon) + \frac{\mu}{2}(t+T_2-\epsilon)^2] \quad , \tag{47}$$

where ϵ=5 msec.

Finally, the signal supplied by the second system loop is

$$C_1 = \alpha_1 \cos[\omega_0(t+T_2-2\epsilon) + \frac{\mu}{2}(t+T_2-2\epsilon)^2]$$

$$+ \beta_1 \sin[\omega_0(t+T_2-2\epsilon) + \frac{\mu}{2}(t+T_2-2\epsilon)^2] \quad . \tag{48}$$

In Eqs. (46) through (48), T_2 is an onset delay that is introduced to permit all reference signals to assume their active states, and for the present case T_2=30 msec. The arguments on the right hand sides of Eqs. (46) and (48) can be brought into coincidence with that of Eq. (47) if the gain parameters are assumed to take the form

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$$\alpha_0 = A_0(t)\ \cos[\mu\varepsilon(t+T_2) + \omega_0\varepsilon - \frac{\mu\varepsilon^2}{2}]$$

$$\beta_0 = A_0(t)\ \sin[\mu\varepsilon(t+T_2) + \omega_0\varepsilon - \frac{\mu\varepsilon^2}{2}]$$

$$\alpha_1 = A_1(t)\ \cos[\mu\varepsilon(t+T_2) + \omega_0\varepsilon - \frac{3}{2}\mu\varepsilon^2]$$

$$\beta_1 = A_1(t)\ \sin[\mu\varepsilon(t+T_2) + \omega_0\varepsilon - \frac{3}{2}\mu\varepsilon^2]\ . \tag{49}$$

In Eqs. (49), $A_0(t)$, $A_1(t)$ are assumed to be constants or slowly varying quantities. Thus, the gain parameters associated with the first two loops vary as phase shifted sinusoids at a radian frequency equal to $\mu\varepsilon$.

In the case under consideration

$$\frac{\mu}{2} = 2\pi \cdot \frac{W_1}{T_3}\ , \tag{50}$$

where W_1 is 500 Hz and T_3 is 128 msec. Thus, for ε=5 msec, the frequency associated with the gain parameters is

$$\frac{\mu\varepsilon}{2\pi} = \frac{2W_1\varepsilon}{T_3} = \frac{5}{128} \times 10^3 \text{ Hz} = 39.0625 \text{ Hz}. \tag{51}$$

The period associated with this frequency is 25.6 msec and a careful examination of the data over time intervals removed from the origin indicate that the gain parameters for the first two

loops vary at a frequency of 39.0625 Hz. Moreover, after the parameters assume a substantially sinusoidal character, α_0 and β_0 assume a quadrature relationship as do α_1 and β_1. However, the phase angles of the waveforms do not agree with the phase angles prescribed by Eqs. (49). For example, the computer generated values for α_0 have zeroes that occur approximately 7.6 msec after the zeroes of the first of Eqs. (49). However, the computer generated values for α_1 occur about 5.2 msec after the zeroes of the third of Eqs. (49). The sum of the two delays as cited above is one half the period of the sinusoidal variation described by the gain parameters. However, the significance that should attach to this is presently unknown and indeed the exact causes of the observed phase misalignments are not understood. The net result of the phase shifts limits the nulling action on the error signal such that the rectified and averaged error signal is about one half the value that would be realized with the received signal alone.

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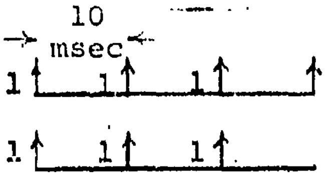
d. Case 4

The signal used in this case consisted of three, unit level components in exact time registry with the first three reference signals, respectively. The component in registry with $R_0(t)$ was assigned a Doppler corresponding to - 2 Kts at 25 kHz. The component coincident with $R_1(t)$ was assigned zero Doppler, and the component coincident with $R_2(t)$ was assigned a Doppler corresponding to +2 Kts at 25 kHz. Doppler was computed as a frequency shift, Δf, from the equation

$$\Delta f = 2f_0 \frac{\Delta \nu}{c} \tag{52}$$

where f_0 was assumed to be the center of an operating band at 25 kHz, $\Delta \nu$ is a radial component of velocity, and c is the propagation speed of sound taken to be 4950 ft/sec. Evaluation of Eq. (52) will show that with $\Delta \nu$=2 Kts, Δf becomes 34 Hz. Because the assumed center of the operating band would be heterodyned to the center of the Echo's Cancellers operating band at 1 kHz, +2 Kts of Doppler would displace a spectral line normally at 1 kHz to 1.034 kHz. Within the signal generator subroutine of the computer program, Doppler is assigned by scaling the time parameter within the program. Thus, for +2 Kts of Doppler, time is scaled by 1.034 and for -2 Kts of Doppler, time is scaled by the factor 0.966. The response of the system to the three component signal is represented in Fig. 9.

The reference signals associated with α_0, β_0, and α_2, β_2 are in time coincidence, at least initially, with the two Dopplered components of the received signal. During earlier portions



Reference Onset Distribution

Signal Component Onset Distribution

RUN CONDITIONS

<u>Component S_0</u>

In time registry with R_0, of unit level and -2Kts Doppler referred to 25 kHz

<u>Component S_1</u>

In time registry with R_1, of unit level and zero Doppler

<u>Component S_2</u>

In time registry with R_2, of unit level and +2Kts Doppler referred to 25 kHz

T = .004 sec

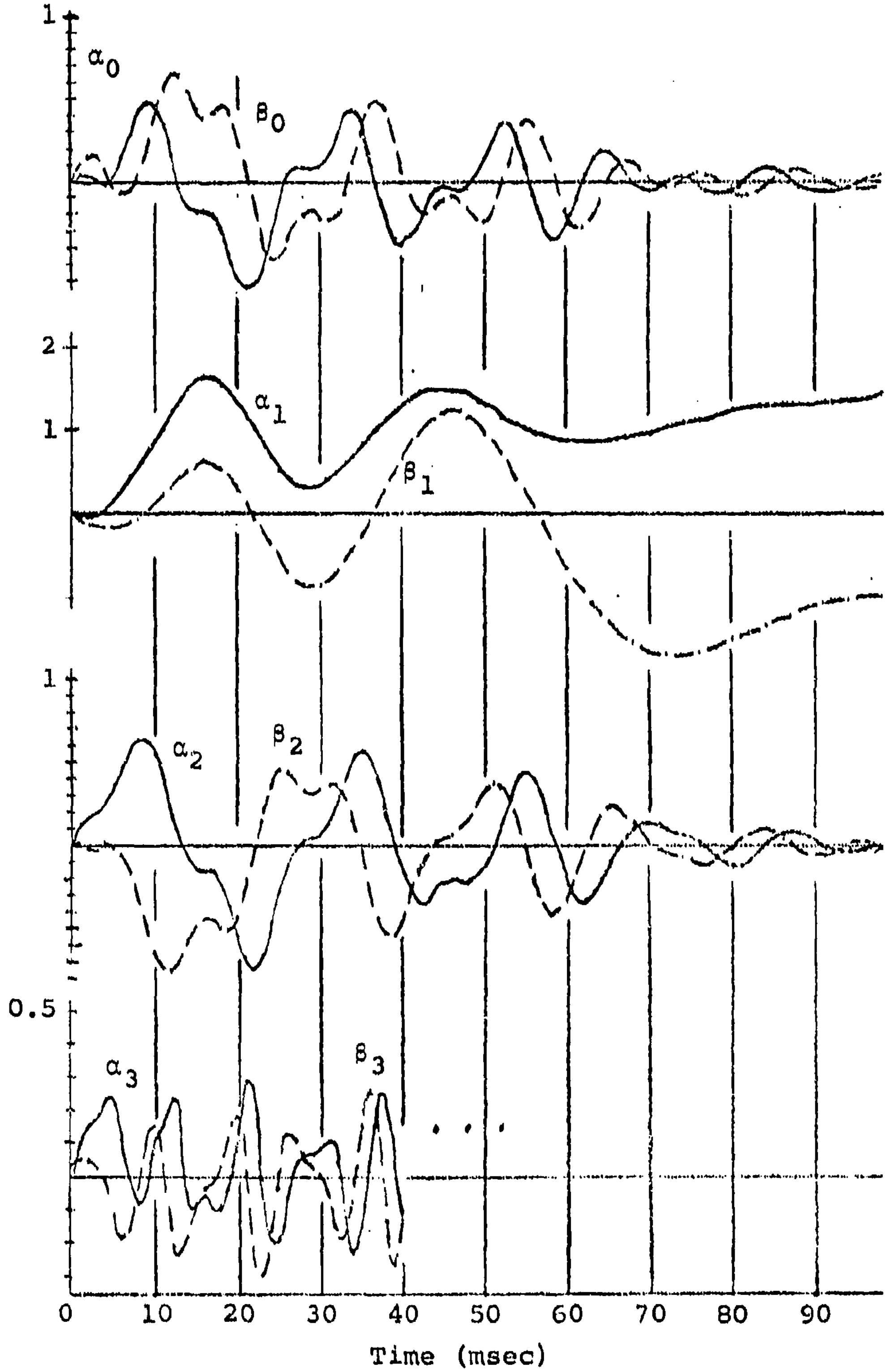


Fig. 9. Gain Parameters Versus Time for a Four Channel, Quadrature Loop System. Case 4.

of the run, these parameters describe relatively wide ranging variations in response to the error signal, but as time progresses the nominal amplitude excursions of the parameters becomes progressively smaller, and toward the end of the run, the small excursions of these parameters indicates that correspondingly small levels of their associated reference functions are being admitted to the error signal channel.

The reference function $R_1(t)$ is in exact time registry with the second, zero-Doppler component of the signal. The gain parameter, α_1, can be seen to rise from zero and increase toward positive values. Ideally, this parameter would approach unity to null the second signal component. However, as can be seen in Fig. 9, α_1 overshoots the value unity and thereafter describes excursions about unity in response to the composite error signal. Thus, α_1 can be viewed as having developed a bias toward the value that causes a null of the second signal component, and superimposed on this bias are additive variations representing a response to the non-zero components retained in the error signal.

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E. DISCUSSION OF RESULTS

The cases considered indicate that each of the gain parameters shows some degree of response to the non-zero components of the error signal. The study has been restricted to cases involving FM signals. For this type of signal and for cases of zero Doppler one can also say that each loop shows a preferential response to those signal components in closest time registry with its associated reference functions. The basis of the preferential response appears to reside in the fact that the gain parameters must describe a sinusoidal variation to null any component with mismatched time registry, that the frequency of the variation increases directly with the degree of error in time coincidence, and that each loop responds preferentially to lower frequencies. There exists a basis for considering each loop to have the frequency response of a single pole, low pass filter with a time constant of 2T, where T is the time constant of the system's integrators. For a four msec time constant, the corresponding cut-off frequency of the low pass filter would be about 19.9 Hz.

In the case involving a single component mismatched by 5 msec from "nearest neighbor" reference signals, the required frequency of gain parameter variation was in excess of 39 Hz. This frequency is also above the cut-off frequency of 19.9 Hz, and difficulties involving phase angle were encountered in the case in question. However, the behavior of the system is not yet understood in a quantitative sense.

In cases involving Doppler, the system behavior becomes more complex, because in this case the frequency requirements imposed by a temporal mismatch between a signal component and a given reference function can be either increased or reduced by the Doppler associated with the signal component.

III. PROPOSED WORK FOR THE NEXT QUARTER

Work during the next quarter will include a consideration of additional cases involving a reduction in the system's delay line parameter, τ. The balance of the effort during the next quarter will be devoted to the preparation of a final report.